Effect of Gravity Waves on Shear Instability in Compound Channels

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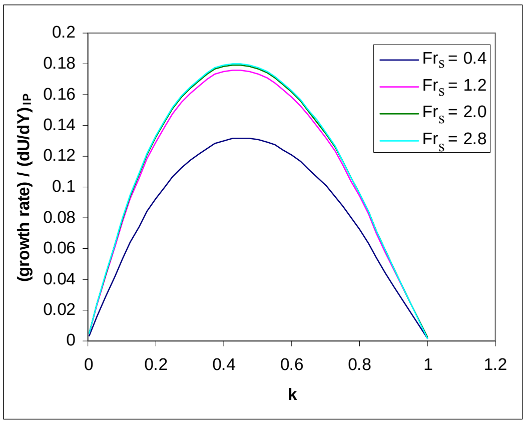
Abstract: The horizontal coherent structures in compound channels can cause momentum exchange between the main channel and the floodplain. Such exchange results in a lowering of the total discharge capacity of a compound channel, compared to the case where the channel and the floodplain are considered separately (van Prooijen 2004). An understanding to the conditions under which such coherent structures form is important for flood prevention. The effect of surface waves on the shear instability leading to the formation of the horizontal coherent structures in compound channels is being investigated. The author suggests that for small bed friction, the only important Froude number is the convective Froude number, Fr_c, defined with the velocity being the velocity difference across the compound channel. When the bed friction becomes large, the instability of the shear layer depends on two Froude numbers: (1) the aforementioned Fr_c, and (2) the stream Froude number, Fr_s, defined with the velocity along the stream. To this end, a linear stability analysis is performed numerically with a shooting procedure in order to account for the wave radiation from the shear layer. The numerical code is validated against previous results in Chu 2010. Then, initial investigation is performed, showing signs of dependence on Fr_s under sufficient bed friction coefficient for Kelvin-Helmholtz modes under low Fr_c (figure 1). The future work will be on (1) finding scaling parameters which correspond to the water depth gradient across the shear layer, (2) choosing the boundary conditions suitable for roll waves and (3) increasing Fr_c.



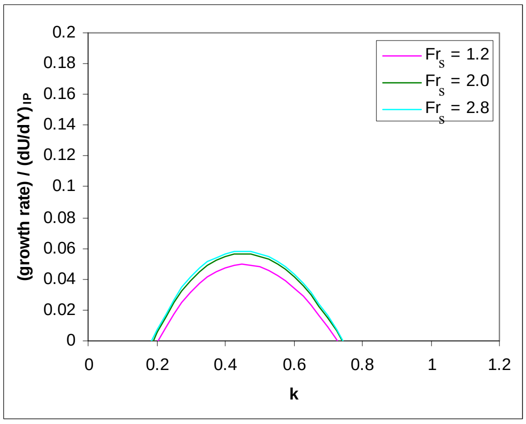


Figure 1. Linear stability curves for compound channels, Fr_c=0.2, Fr_s=0.4~2.8. (a, left) no bed friction S=0.0, and. (b, right) under a bed friction S=0.08, with S being the stability parameter. For S=0.0, the difference in growth rate curves between Fr_s=0.4 and Fr_s=1.2 is likely to be caused by the water depth gradient across the shear layer. For S=0.08, the flow with Fr_s=0.4 is already stable for Kelvin-Helmholtz modes. The distance between curves Fr_s=1.2 and Fr_s=2.0 in the figure (1a) and (1b) shows the effect of stream Froude number on the shear instability. The growth associated with the roll wave at high Fr_s is not captured currently.

Though compound channel flows with moderate to high Froude numbers are not common, a high Froude number compound channel flow can occur in peak flows with a long return period (e.g. 100 year). Hence, the current work has significant practical merit since most open channels are designed for peak discharges (Kolyshkin and Ghidaoui 2002).

Reference:

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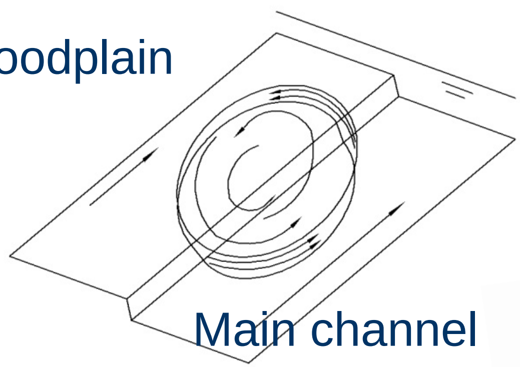
Kolyshkin, A. A. and Ghidaoui, M. S. (2002). Gravitational and shear instabilities in compound and composite channels, *J. Hydr. Engrg*., ASCE, **128**(**12**). pp 1076-1086

Effect of gravity waves on shear instability in compound channels

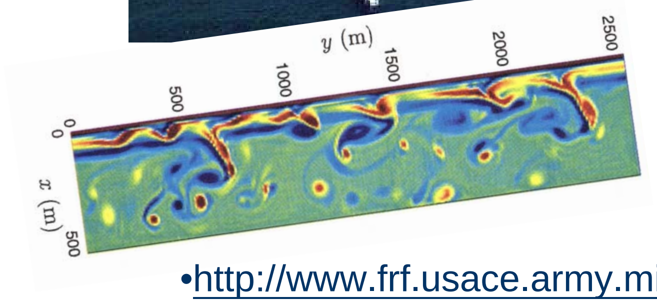
M. Y. Lam and M. S. Ghidaoui

Introduction

- Shear flows under moderate to high Froude number
- Two dimensional coherent structures (2DCS)

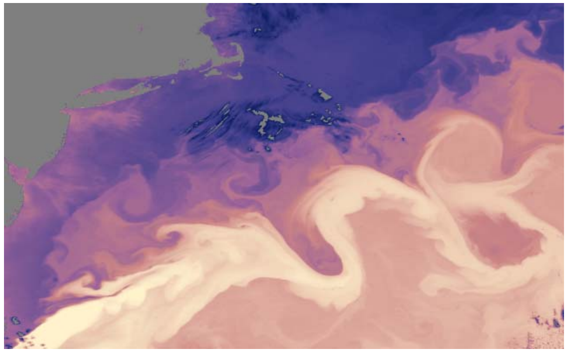


•van Prooijen 2004



•http://www.frf.usace.army.mil/aboutUS/facilities.shtml

•Ozkan-Haller and Kirby 1999



•http://earthobservatory.nasa.gov/IOTD/view.php?id=5432

Introduction

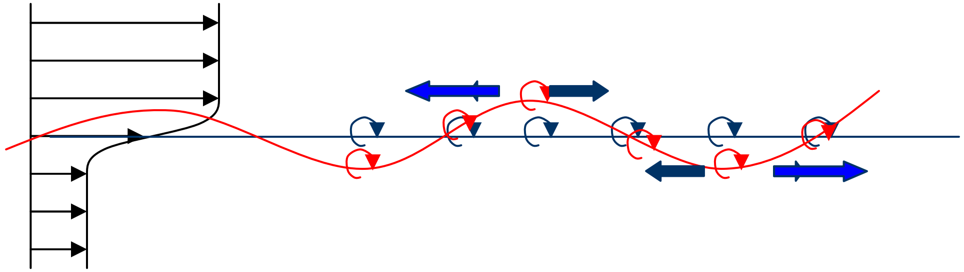
http://www.greenpower.org.hk/river/chi/sm_human.asp

http://redac.eng.usm.my/html/projects/ESG/ESG.html

- The current work devotes to the effects of Froude number on shear instability of a half of a compound channel.
  - 2DCS reduces discharge capacity (van Prooijen 2004), so it is important in flood modeling.
  - A prototype for other flows, e.g. shallow jets and wakes are two shallow mixing layers (Tenneke and Lumly 1972).

Generation of coherent structures

- Kelvin Helmholtz (KH) instability



$$S = \frac{c_f \delta}{2\tilde{H}_{IP}} \frac{\tilde{U}_{IP}}{\Delta U}$$

Large S: Fluctuations decay,
return to parallel flow
(stable)

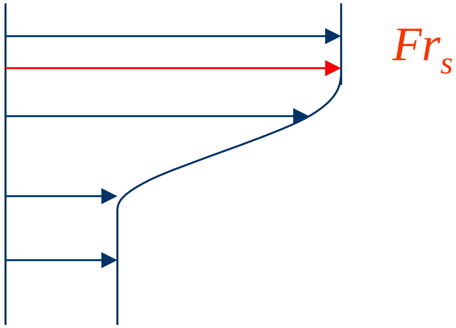
Small S: Fluctuations grow,
coherent structure develops
(unstable)

Effect of Froude number

- Froude number is shown to have a stabilizing effect by allowing the fluctuation energy to be stored and transmitted in a different form (gravity waves), although the Froude number is defined differently:
  - Stream Froude number (Falques and Iranzo 1994, Ghidaoui and Kolyshkin 1999, Ghidaoui et al 2009). The effect is closely related with roll waves when Fr_s>2.

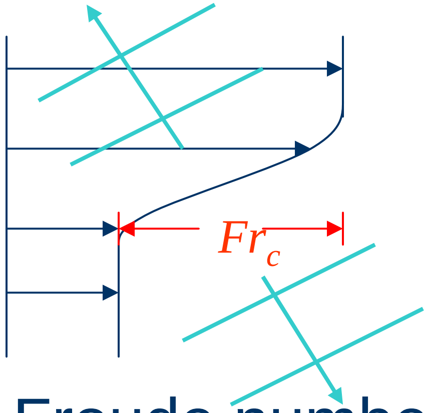
Roll waves in Qian Tang Jiang, a river in Zhe Jiang Province, China

http://tw.people.com.cn/BIG5/14810/14860/942260.html



Effect of Froude number

– Convective Froude number (Chu 2010, Pinilla and Chu 2010). Related to wave radiation from the shear layer, taking away energy available for Kelvin-Helmholtz instability.



- The respective effects of the two Froude numbers on the shear instability are unclear, since previous researches considers only one Froude number at a time.

Effect of Froude number

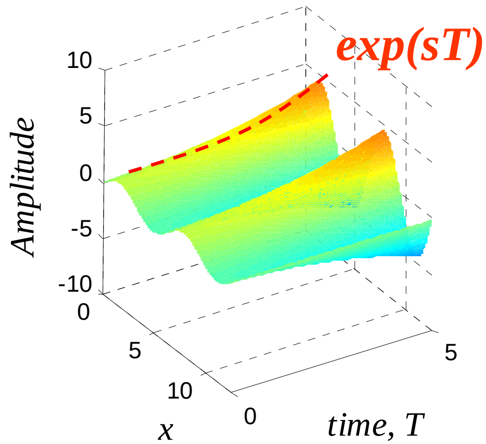
- Propose hypothesis on the effects of the two different Froude numbers:
  - Small bed friction:

    Generation of 2DCS depends on Fr_c only, since the formation of wave radiation does not need bed friction.
  - Sufficient bed friction:

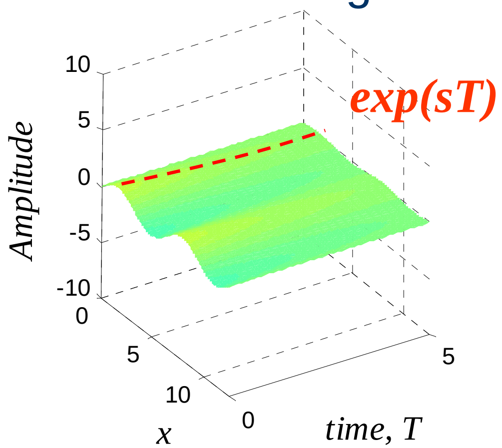
    Generation of 2DCS depends on both Fr_s and Fr_c, since roll wave needs bed friction to form.

Methodology

- The current study compares the growth rate of the fluctuations which give rise to the coherent structures under different hydraulic parameters.
- Linear stability analysis:
  - Determines whether the fluctuations putting on top of a particular base flow will grow or decay (i.e. determine s - the growth rate).



Growth (positive s): leads to 2DCS



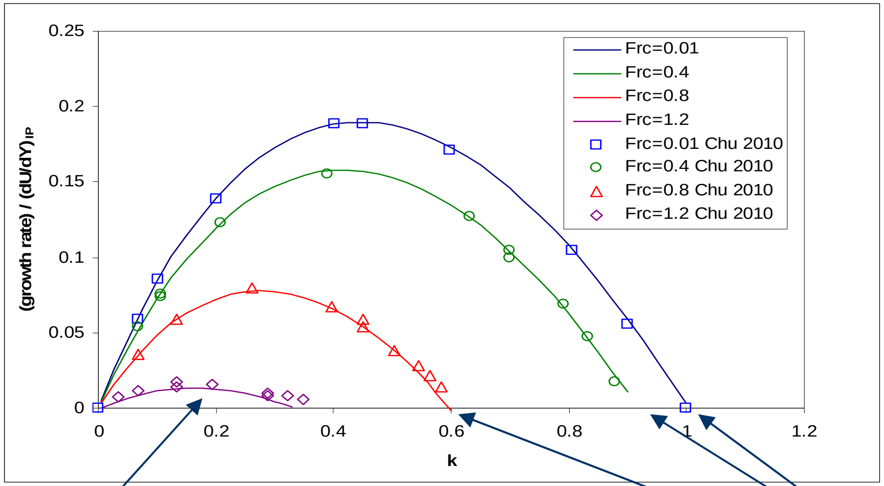
Decay (negative s): returns to parallel flows

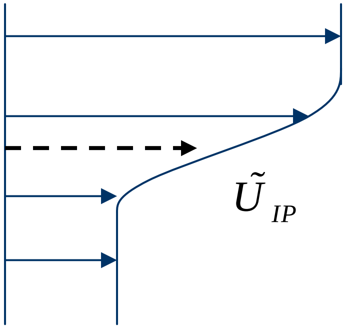
Methodology

- Linear stability analysis (cont'd)
  - Determine s by the solution of linearized shallow water equations with some boundary conditions.
  - Wave radiation under high Fr_c causes waves to shoot to the infinity in cross-stream direction.
  - Violates the boundary conditions used in previous research (Ghidaoui and Kolyshkin 1999, Kolyshkin and Ghidaoui 2002) , hence another boundary conditions are derived.
  - For the numerical solution, a shooting procedure is adopted (e.g. Jackson and Grosch 1989).

Validation

- Comparison with nonlinear numerical simulations of shallow mixing layers in Chu 2010:





Wavespeed does not equal to $\tilde{U}_{IP}$

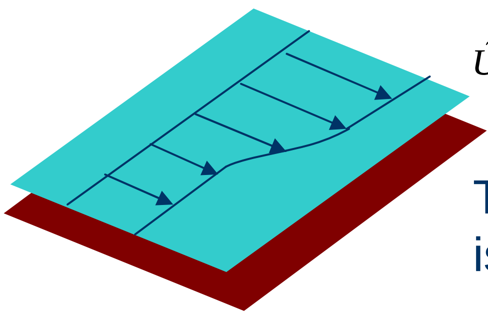
Wave radiation modes

Wavespeed = $\tilde{U}_{IP}$

Kelvin-Helmholtz modes

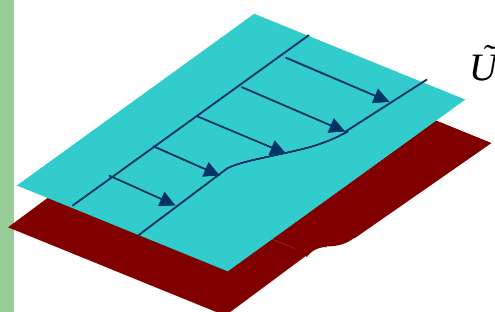
Initial results

- Base flow: (1) shallow mixing layers and (2) compound channels



$$\tilde{U}(Y) = \tilde{U}_{IP} + \frac{\Delta U}{2}\tanh(\frac{2Y}{\delta})$$

The cross section $\tilde{H}(Y)$ is constant.



$$\tilde{U}(Y) = \tilde{U}_{IP} + \frac{\Delta U}{2}\tanh(\frac{2Y}{\delta})$$

The cross section $\tilde{H}(Y)$ is defined so that Fr_s is constant across the channel.

$$Fr_c = \frac{\Delta U}{\sqrt{g\tilde{H}_1} + \sqrt{g\tilde{H}_2}}$$ =0.2

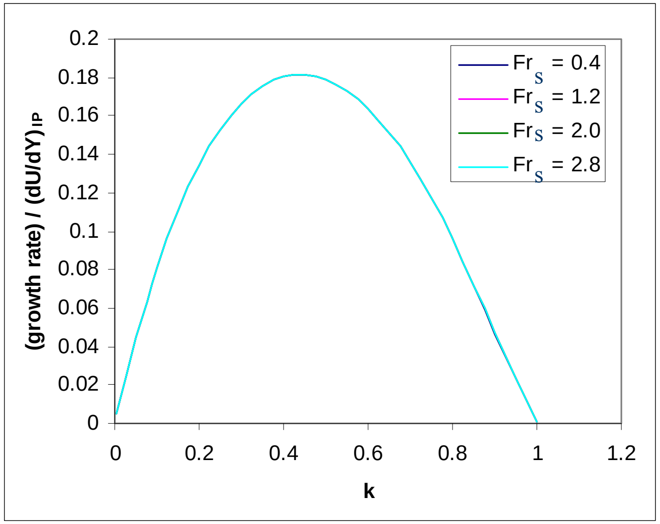
$$Fr_s = \frac{\tilde{U}_{IP}}{\sqrt{g\tilde{H}_{IP}}}$$ (Case 1), or

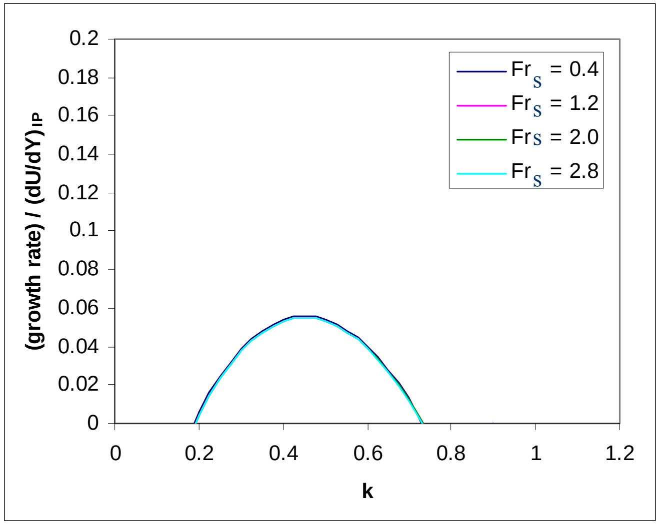
$$Fr_s = \frac{\tilde{U}(Y)}{\sqrt{g\tilde{H}(Y)}}$$ (Case 2)

=0.4, 1.2, 2.0, 2.8

$$S = \frac{c_f \delta}{2\tilde{H}_{IP}} \frac{\tilde{U}_{IP}}{\Delta U}$$ =0.0, 0.08

Initial results: shallow mixing layers



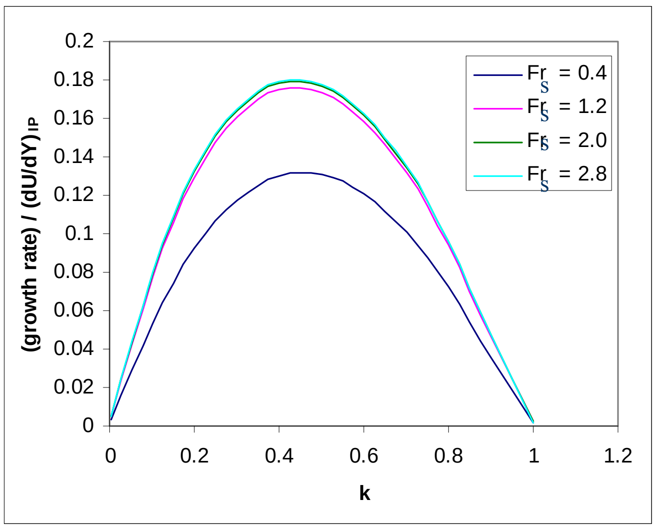


S=0.0, Fr_c=0.2

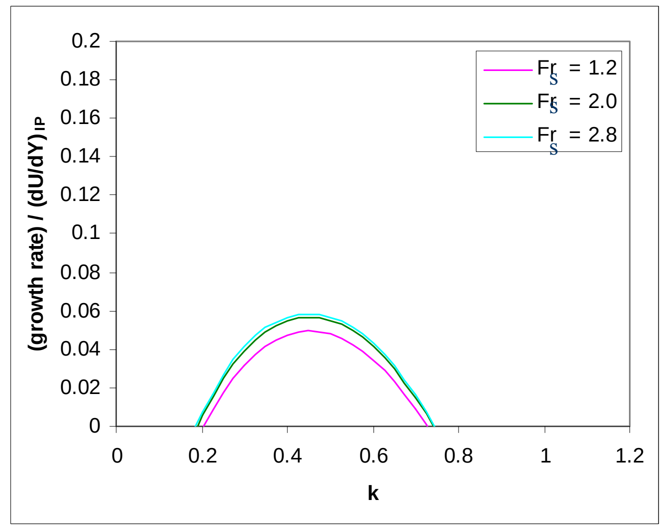
S=0.08, Fr_c=0.2

- Independent of stream Froude number for both cases, with and without bed friction. The case no bed friction agrees Pinilla and Chu 2009, the case with bed friction seems disproving the current hypothesis.
- No roll wave is observed for S=0.08, $Fr_s \geq$2.0, likely to due to the boundary condition used.
- Roll waves can be the governing instability, overriding the KH instability, if the current code can capture it (Kolyshkin and Ghidaoui 2002).

Initial results: compound channels



S=0.0, Fr_c=0.2



S=0.08, Fr_c=0.2

- For $S=0$, $Fr_s=0.4$ does not collapse with other results, likely to be due to bottom slope. However, the attempt of scaling with the momentum gradient, $\frac{1}{\tilde{H}_{IP}}\left(\frac{\partial \tilde{U}\tilde{H}}{\partial Y}\right)_{IP}$, failed.
- The distance between curves Fr_s=1.2 and Fr_s=2.0 in the two figures shows the effect of stream Froude number on the shear instability under the effect of water depth gradient.

Conclusion and future work

- The effect of Froude number on the shear instability in compound channels is being investigated. A numerical code is developed to capture the wave radiation under the effect of bed friction. Evidence of dependence on stream Froude number under sufficient bed friction is observed, but seems require water depth gradient. The roll wave, being crucial to the effect of Fr_s, is not yet captured.

- The future work will be on (1) choosing the boundary conditions suitable for roll waves, (2) studying the effect of water depth gradient and (3) increasing Fr_c.

- The separation of two Froude numbers Fr_s and Fr_c, suggests:
  - The validity of the rigid-lid assumptions used in stability analysis (e.g. Chu et al 1991, Chen and Jirka 1997) and numerical simulations (van Prooijen 2004, Hinterberger et al 2007) may depend on two Froude numbers.
  - One may need to consider two Froude numbers in compound and composite channel design.
  - Previous results on the effect of Froude number may need re-interpretation.

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